Exam Statistical Physics **11-4-2011**

Problem 1

a) Explain what is meant with the concepts *macrostate* and *microstate*.

b) Give the two main postulates of statistical physics.

c) Describe in a short paragraph (100-200 words) the differences between the *microcanonical* ensemble, the *canonical* ensemble and the *grand canonical* ensemble (e.g. what are the typical quantities involved, to what situations the three concepts pertain etc.)

Problem 2 (= exercise 3.2)

A magnetic specimen consists of N atoms, each having a magnetic moment μ.
In an applied homogeneous magnetic field $\mathcal{H}$ each atom may align itself parallel or antiparallel to the field, the energy of an atom in the two orientations being respectively

$$\varepsilon_{\pm} = \mp\left(\mu\,\mu_0\mathcal{H} + \frac{k\theta\,\mathcal{M}}{\mu N}\right).$$

Here θ is a temperature characteristic for the specimen, $\mathcal{M}$ is its magnetic moment, and μ_0 is the permeability of the vacuum.

a) Show that the magnetic moment of the specimen at temperature T is given by

$$\frac{\mathcal{M}}{\mu N} = \tanh\left[\frac{1}{kT}\left(\mu\,\mu_0\mathcal{H} + \frac{k\theta\,\mathcal{M}}{\mu N}\right)\right].$$

b) If $\mathcal{M} << \mu N$, obtain the temperature dependence of the magnetic susceptibility per atom, $\chi = \dfrac{\mathcal{M}}{\mathcal{H} N}$ (in terms of μ, μ_0, k, T and θ).

Problem 3

a) Explain the meaning of the symbols in the equation of Clausius-Clapeyron:

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V}$$

b) A pressure cooker works at a pressure P = 2 bar.
Assume that at any time the water is in equilibrium with its vapour and that the water vapour can be considered as an ideal gas.
The boiling point of water for P = 1 bar is given by $T_b = 373$ K.
The enthalpy of vaporization of water is given by ΔH = 40,6 kJ/mol and independent of temperature and pressure.
Determine the temperature in the pressure cooker.

Problem 4

a) Show that the partition function of a photon gas can be written as:

$$Z_{ph} = \prod_{r=1}^{\infty} \frac{1}{1-e^{-\beta\varepsilon_r}}$$

in which $\beta = \dfrac{1}{kT}$, with k Boltzmann's constant and T the temperature.

b) Show that the free energy of a photon gas is given by: $F = -\dfrac{\pi^2 k^4 T^4 V}{45 c^3 \hbar^3}$,

where V is the volume, c is the velocity of light, and $\hbar = \dfrac{h}{2\pi}$ with h the constant of Planck.

Hint: Remember that the density of states for a particle in momentum space is given by $f(p)dp = V\dfrac{4\pi\, p^2 dp}{h^3}$ *and adapt for photons of frequency* ω.

c) Give an expression for the pressure P of the photongas in terms of k, T, c and $\hbar$.

Problem 5

Consider a two-dimensional gas of non-relativistic electrons.

a) Make plausible that the density of states in momentum space is given by $f(p)dp = \dfrac{4\pi\, A\, p\, dp}{h^2}$, in which A is the area to which this two-dimensional gas is constrained.

b) Show that the number of particles N can be written as:

$$N = \frac{A\, 4\pi\, m}{h^2} \int_0^{\infty} \frac{d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1}$$

In this expression is ε the energy of an electron, μ the chemical potential, and m the mass of an electron.

c) Show that the Fermi energy is given by: $\varepsilon_F = \dfrac{h^2 N}{4\pi\, m A}$

d) Calculate for $T = 0$ the average energy of an electron in terms of the Fermi energy.

Physical constants:

Getal van Avogadro: $N_A = 6{,}02 \times 10^{23}\ mol^{-1}$

Constante van Planck: $h = 6{,}626 \times 10^{-34}$ Js

$$\hbar = \frac{h}{2\pi} = 1{,}055 \times 10^{-34} \text{ Js}$$

Constante van Boltzmann: $k = 1{,}381 \times 10^{-23}\ J\ K^{-1}$

Gasconstante: $R = 8{,}315\ J\ mol^{-1}\ K^{-1}$

Lichtsnelheid: $c = 3 \times 10^{8}\ m\ s^{-1}$

Rustmassa elektron $m_e = 9{,}11 \times 10^{-31}$ kg

Rustmassa proton $m_p = 1{,}67 \times 10^{-27}$ kg

Bohr magneton $\mu_B = 9{,}27 \times 10^{-24}\ A\ m^2$

Integrals:

n	$\int_0^\infty dx\, x^n e^{-ax^2}$ (a > 0)	$\int_0^\infty \frac{x^n dx}{e^x - 1}$	$\int_0^\infty \frac{x^n dx}{e^x + 1}$	$\int_0^\infty \frac{x^n e^x}{(e^x - 1)^2}$	$\int_0^\infty x^n \ln(1 - e^{-x}) dx$
0	$\frac{1}{2}\sqrt{\left(\frac{\pi}{a}\right)}$	diverges	ln 2	diverges	$-\frac{\pi^2}{6}$
1/2	$\frac{0{,}6127}{a^{3/4}}$	$2{,}612\frac{\sqrt{\pi}}{2}$	0,6781	diverges	$-1{,}341\frac{\sqrt{\pi}}{2}$
1	$\frac{1}{2a}$	$\frac{\pi^2}{6}$	$\frac{\pi^2}{12}$	diverges	-1,202
3/2	$\frac{0{,}4532}{a^{5/4}}$	$1{,}341\frac{3\sqrt{\pi}}{4}$	1,153		$-1{,}127\frac{3\sqrt{\pi}}{4}$
2	$\frac{1}{4a}\sqrt{\frac{\pi}{a}}$	2,404	1,803	$\frac{\pi^2}{3}$	$-\frac{\pi^4}{45}$
5/2	$\frac{1{,}662}{a^{7/4}}$	$1{,}127\frac{15\sqrt{\pi}}{8}$	3,083		-3,505
3	$\frac{1}{2a^2}$	$\frac{\pi^4}{15}$	$\frac{7\pi^4}{120}$	7,212	-6,221
7/2	$\frac{0{,}5665}{a^{9/4}}$	12,268	11,184		
4	$\frac{3\sqrt{\pi}}{8a^{5/2}}$	24,886	23,331	$\frac{4\pi^4}{15}$	